

TESTS OF GRAVITY AT THE SOLAR SYSTEM SCALE

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As confirmed by tests performed in the solar system, General Relativity (GR) presently represents the best description of gravitation. It is however challenged by observations at very large length scales, and already at the solar system scale, tracking of the Pioneer 10/11 probes has failed to confirm their expected behavior according to GR. Metric extensions of GR, which are presented here, have the quality of preserving the fundamental properties of GR while introducing scale dependent modifications. We show that they moreover represent an appropriate family of gravitation theories to be compared with observations when analysing gravity tests. We also discuss different tests which could allow one to determine the metric extension of GR prevailing in the solar system.

1 Introduction

General Relativity (GR) is unique among fundamental theories as it has first been introduced on the basis of general principles¹, before being confirmed by observations². However, while GR agrees with the most precise observations made in the solar system, recent observations performed at larger length scales show inconsistencies between the visible content of larger parts of the Universe and the gravitation laws according to GR. The anomalous rotation curves of galaxies³ and the anomalous acceleration of type Ia supernovae⁴ can point at the existence of important amounts of dark matter in galactic halos^{5,6} and of dark matter and energy at the cosmological scale^{7,8}. But, should these dark constituents remain unobserved, this could mean that the gravitation laws have to be changed at these scales. The necessity to modify GR may even come earlier, already at the solar system scale, if the anomaly observed on the navigation data of the Pioneer 10/11 probes⁹ did not find a conventional explanation.

Beside observational data, theoretical arguments also plead for considering the possibility of scale dependent gravitation laws. The coupling constants of the other three fundamental interactions are known to develop a scale dependence as a consequence of radiative corrections, a property which justifies the idea of a possible unification of all fundamental interactions. Gravitation, being also both geometry and a field theory, should share this property. Assuming "asymptotic safety"¹⁰, renormalization group techniques allow one to derive the general features of the scale dependence of gravitation. When combined with observational constraints, they lead to favour a family of metric extensions of GR for describing gravitation¹¹.

We briefly review here the properties of such metric extensions of GR and obtain a parametriza-

tion of these theories suiting phenomenological purposes. We discuss how they can be used when analysing gravity tests performed in the solar system and when searching anomalous gravitation properties with respect to GR.

2 General Relativity and its metric extensions

GR plays an exemplary role among fundamental theories because of two essential properties: it describes gravitation both as geometry and as a field theory. The first property deeply affects modern spacetime metrology, which relies on a strong relation between gravitation and geometry: definitions of reference systems depend on an underlying metric $g_{\mu\nu}$ which refers to solutions of gravitational equations of motion¹². This assumption is made possible by the identification of gravitation with the geometry of spacetime. According to GR, all bodies, massive and massless ones as well, follow geodesics in absence of non gravitational forces. Geodesics are obtained from a universal geometric distance, defined by the metric $g_{\mu\nu}$ and which also coincides with the proper time delivered by clocks along their motions. This results in particular in the universality of free fall, a principle which has been verified to hold at very different length scales, ranging from millimeter^{13,14} to astronomic scales¹⁵, and at a very high precision level (10^{-13}).

On the other hand, as one of the four fundamental interactions, gravitation is also described by means of a field, characterized by the way it couples to its sources. In GR, the metric field couples to energy-momentum tensors $T_{\mu\nu}$ through its Einstein curvature $E_{\mu\nu}$, a particular combination of Ricci ($R_{\mu\nu}$) and scalar (R) curvatures. As both tensors are divergenceless, $T_{\mu\nu}$ as a consequence of conservation laws and $E_{\mu\nu}$ of Bianchi identities, coupling can be realized by a unique proportionality constant, Newton gravitation constant G_N ^{16,17} (c is light velocity)

$$E_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G_N}{c^4}T_{\mu\nu} \quad (1)$$

But gravitation is a very weak interaction, so that the particular form of the gravitational equations of motion (1) is extremely difficult to bring to experimental test. Usually, tests of gravity are only performed in an indirect way, by comparing observations with predictions which can be obtained on the basis of metrics satisfying equations (1). As a consequence, the particular field theory characterizing gravitation, and G_N , appear to be tested with much less precision than the geometric nature of gravitation.

Moreover, theoretical arguments suggest that the gravitational equations of motion (1) cannot remain valid over arbitrary energy or length scales. Indeed, as a universal mechanism occurring in field theories, higher order processes modify couplings and propagators. This is the case for electro-weak and strong interactions, whose coupling constants become scale dependent and follow renormalization group trajectories. In a similar way, radiative corrections should lead to a scale dependence of the gravitational coupling, making the gravitational equations specified by GR (1) only approximately valid¹⁰. Remarkably, these theoretical arguments appear to be met by anomalous observations performed at very large length scales^{3,4}, which can also be interpreted as questioning the validity of GR at such scales^{5,6,7,8}.

Although the case of gravitation shows to be theoretically involved, the main features of the expected scale dependences can nonetheless be obtained from general properties. The symmetries, or gauge invariance, underlying gravitation constrain observables to take the form of geometric quantities^{18,19}. Hence, the further couplings induced by radiative corrections involve squares of curvatures so that GR can indeed be seen to be embedded in a family of renormalizable field theories. This implies that, when radiative corrections are taken into account, gravitation can still be described by a metric theory, but that the single gravitation constant G_N must be replaced by several running coupling constants²⁰ characterizing additional terms in the Lagrangian. There results that GR, defined by Einstein-Hilbert Lagrangian (1), is extended

to a theory which is both non local, as a result of radiative corrections, and non linear, due to their geometric nature. It leads to gravitational equations of motion which can be put under a general form, with a susceptibility replacing Newton gravitation constant²¹

$$E_{\mu\nu} = \chi_{\mu\nu}(T) = \frac{8\pi G_N}{c^4} T_{\mu\nu} + \delta\chi_{\mu\nu}(T) \quad (2)$$

The resulting equations appear to be difficult to solve due to a particular mixing realized between non linearity and non locality.

As another general property, radiative corrections can be seen to essentially differ in two sectors corresponding to couplings to massless or massive fields^{20,21}: in the former case, traceless energy-momentum tensors couple to Weyl curvature only, while in the latter case couplings between energy-momentum traces and the scalar curvature also occur. GR should then be extended to metric theories which are characterized by two sectors, of different conformal weights, with corresponding running coupling constants $G^{(0)}$ and $G^{(1)}$ which generalize Newton gravitation constant G_N . The relations between coupling constants can be given simple expressions in a linearized approximation (using a representation of fields in terms of momentum k and introducing the corresponding projectors π on trace and traceless parts)^{22,23}

$$\begin{aligned} E_{\mu\nu} &= E_{\mu\nu}^{(0)} + E_{\mu\nu}^{(1)}, & \pi_{\mu\nu} &\equiv \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \\ E_{\mu\nu}^{(0)} &= \left\{ \pi_\mu^0 \pi_\nu^0 - \frac{\pi_{\mu\nu} \pi^{00}}{3} \right\} \frac{8\pi G^{(0)}}{c^4} T_{00}, & E_{\mu\nu}^{(1)} &= \frac{\pi_{\mu\nu} \pi^{00}}{3} \frac{8\pi G^{(1)}}{c^4} T_{00} \\ G^{(0)} &= G_N + \delta G^{(0)}, & G^{(1)} &= G_N + \delta G^{(1)} \end{aligned} \quad (3)$$

Although the two running coupling constants remain close to G_N , non locality and non linearity combine in an intricate way and do not allow a decomposition as simple as (3) to hold beyond the linearized approximation. Alternatively, one can look for non linear but local theories which approximate the previous metric extensions of GR. It is remarkable that, due to the presence of two sectors, such approximations can be obtained which involve higher order field derivatives and nonetheless correspond to theories with stable ground states²⁴.

To the theoretical difficulties implied by non locality combined with non linearity, some compensation can be found in direct observations. Indeed, the latter show that gravitation should remain very close to GR over a large range of scales. They moreover show that departures from GR can happen not only at large energy scales, as expected if gravitation should unify with other fundamental interactions, but also at large length scales^{3,4}. Gravitation tests performed up to now make it legitimate to consider the effective gravitation theory at ordinary macroscopic length scales to be a perturbation of GR². Solutions of the generalized equations (2) should then correspond to perturbations of the solutions of GR equations of motion (1). Equivalently, equations (2) may be seen as providing metrics which remain close to those determined by GR and just differ from the latter by curvature anomalies²⁵

$$\begin{aligned} E &= [E]_{\text{GR}} + \delta E, & [E]_{\text{GR}} &= 0 & \text{where} & & T \equiv 0 \\ \delta E &= \delta E^{(0)} + \delta E^{(1)} \end{aligned} \quad (4)$$

Metric extensions of GR are thus characterized by two independent components of Einstein curvature tensor $\delta E^{(0)}$ and $\delta E^{(1)}$, reflecting the two different running coupling constants $G^{(0)}$ and $G^{(1)}$ modifying G_N (as seen in the linear approximation (3)). When solving the gravitation equations of motion (2), the two independent Einstein curvature components are replaced by two gauge-invariant potentials Φ_N and Φ_P (for a point-like source, using Schwarzschild coordinates)

$$\delta E_0^0 \equiv 2u^4(\Phi_N - \delta\Phi_P)'', \quad \delta E_r^r \equiv 2u^3\Phi_P' \quad u \equiv \frac{1}{r}, \quad (')' \equiv \partial_u \quad (5)$$

In the case of GR, Einstein curvature vanishes and the solution depends on a single potential Φ_N taking a Newtonian form (Φ_P vanishing in this case). In the general case, the potential Φ_N extends Newton potential while Φ_P describes a second gravitational sector. These potentials can be seen as a parametrization of admissible metrics in the vicinity of GR solutions, which thus represent good candidates for extending GR beyond ordinary macroscopic scales. This parametrization appears to be appropriate for analysing existing gravity tests and confronting GR with plausible alternative theories of gravitation.

3 Phenomenology in the solar system and gravity tests

The solution of the gravitation equations of motion (2) takes a simple form in the case of a stationary point-like gravitational source, as it corresponds to a static isotropic metric which reduces to two independent components (written here in spherical isotropic coordinates)

$$\begin{aligned} ds^2 &= g_{00}c^2dt^2 + g_{rr}\left(dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right) \\ g_{00} &= [g_{00}]_{\text{GR}} + \delta g_{00}, \quad g_{rr} = [g_{rr}]_{\text{GR}} + \delta g_{rr} \end{aligned} \quad (6)$$

$[g]_{\text{GR}}$ denotes the approximate metric satisfying GR equations of motion (1), which can be written in terms of Newton potential. In this case, the two independent components of the metric δg_{00} and δg_{rr} are in one to one correspondence with the two independent components of Einstein curvature $\delta E^{(0)}$ and $\delta E^{(1)}$. Quite generally, the explicit expressions of the metric components in terms of the two gravitational potentials Φ_N and Φ_P (5) are obtained by inverting the usual relation between metrics and curvatures²⁵.

The most precise tests of GR have been realized in the solar system. Phenomenology in the solar system is usually performed with parametrized post-Newtonian (PPN) metrics^{26,27}. Neglecting the Sun's motions, the corresponding PPN metrics reduce to the form (6), with g_{00} and g_{rr} being determined by Newton potential ϕ and two Eddington parameters β and γ

$$\begin{aligned} g_{00} &= 1 + 2\phi + 2\beta\phi^2 + \dots, \quad g_{rr} = -1 + 2\gamma\phi + \dots \\ \phi &\equiv -\frac{G_N M}{c^2 r}, \quad |\phi| \ll 1 \end{aligned} \quad (7)$$

The parameters β and γ describe deviations from GR (obtained for $\beta = \gamma = 1$) in the two sectors, corresponding respectively to effects on the motion of massive probes and on light deflection. PPN metrics are a particular case of metric extensions of GR, corresponding to a two-dimensional family which describes non vanishing but short range Einstein curvatures

$$\begin{aligned} \Phi_N &= \phi + (\beta - 1)\phi^2 + O(\phi^3), \quad \Phi_P = -(\gamma - 1)\phi + O(\phi^2) \\ \delta E_0^0 &= \frac{1}{r^2}O(\phi^2), \quad \delta E_r^r = \frac{1}{r^2}\left(2(\gamma - 1)\phi + O(\phi^2)\right) \quad [\text{PPN}] \end{aligned} \quad (8)$$

In contrast, general metric extensions of GR are parametrized by two gravitational potentials Φ_N and Φ_P (5) describing arbitrary Einstein curvatures. These two functions may be seen as promoting the constant parameters β and γ to scale dependent functions. The latter manifest themselves as an additional dependence of gravitational effects on a geometric distance. The latter can be either a distance between points (as the probe and the gravitational source) or a distance between a point and a geodesic (as the impact parameter of a light ray).

Existing gravity tests put constraints on possible deviations from GR, hence on allowed metric extensions of GR (6) at the scale of the solar system. Direct scale dependence tests have up to now been performed in the first sector only. They were designed to look for possible modifications of Newton potential taking the form of a Yukawa potential ($\delta\phi(r) = \alpha e^{-\frac{r}{\lambda}}\phi(r)$), characterized by a strength parameter α and a range λ . These tests, performed for λ ranging from

the submillimeter range, using dedicated experiments^{28,29}, to the range of planetary orbits, using probe navigation data and planetray ephemerides^{28,30}, show that the strength α of a Yukawa-like perturbation must remain rather small at all these scales, so that the form of the gravitational potential in the first sector is rather strongly constrained to remain Newtonian. However, constraints become much less stringent below the submillimeter range, where Casimir forces become important²⁹, and at scales of the order of the outer solar system, where observations used to determine ephemerides become less precise. They moreover only concern the first sector.

The increasing set of observations performed in the solar system has progressively reduced the allowed deviations from GR for the two PPN parameters β and γ . Presently, the best constraint on the value of γ is given by the measurement of the Shapiro time delay, induced by the gravitational field of the Sun on the radio link which was used to follow the Cassini probe during its travel to Saturn³¹. GR prediction for the variation of the deflection angle, near occultation by the Sun, has been confirmed, constraining γ to be close to 1 with a precision of 2.5×10^{-5} . A similar bound is provided by VLBI measurements of light deflection². One may remark that such a precision is obtained when assuming that the parameter γ remains constant. As the deflection angle decreases with the impact parameter of the ray, the precision on the measurement of the deflection angle is mainly due to small impact parameters. As a result, the corresponding constraints should be sensitively less stringent when confronted to general metric extensions of GR, which allow γ to depend on the impact parameter of the ray.

The value of γ being assumed, the parameter β can be obtained either by means of a direct measurement, such as Lunar Laser Ranging¹⁵, measuring the Sun polarization effect on the Moon orbit around the Earth, or by means of big fits, using all data made available by probe navigation and astrometry measurements, to determine planet ephemerides^{32,33,34}. Both methods lead to similar constraints on β , fixing the latter to remain close to 1, up to deviations less than 10^{-4} . Let us remark that these determinations are performed at the scale of the Moon orbit in one case, and at a scale of several astronomic units (AU) in the other case. They can also be considered as independent estimations of β being performed at different length scales.

Available data for gravitation at large length scales in the solar system are rather few. Hence, the navigation data of the Pioneer 10/11 probes, during their travel in the outer part of the solar system, provide an important consistency check for models of gravitation in the solar system. Remarkably, the analysis of Doppler data has failed to confirm the predictions made according to GR. Comparison of observed with predicted values resulted in residuals which did not vanish but could be interpreted as exhibiting the presence of an anomalous acceleration $a_P = (0.87 \pm 0.13) \text{ nm s}^{-2}$, directed towards the Sun or the Earth, and approximately constant over distances ranging from 20AU to 70AU⁹. Many attempts have been made to find a conventional explanation to the Pioneer anomaly as a systematic effect either related to the probe itself, allowed by a loss of energy from power generators on board, or to the environment of the probe, due to the presence of dust or gravitating matter in the outer solar system³⁵. These have been followed by sustained efforts for recovering further data and performing new analyses covering the whole Pioneer 10/11 missions³⁶. Up to now, these attempts have remained unsuccessful in explaining the totality of the Pioneer anomaly.

Furthermore, a recent study, confirming the secular part of the Pioneer anomaly, has also analysed the modulations apparent in the Doppler data, showing that their frequencies correspond to the Earth's motions, and that the Doppler residuals can be further reduced by introducing simple modulations of the radio links³⁷. Modulated anomalies cannot be produced by a conventional explanation of the secular part but require a further mechanism (trajectory mismodeling, solar plasma effects, ...) to be accounted for. On the other hand, simple models modifying the metric are able to reproduce both types of anomalies. These features leave the possibility of a common gravitational origin of the Pioneer anomalies, pointing at a deficiency of GR occuring at length scales of the order of the solar system size.

4 Tests of metric extensions of GR

Besides being favoured by theoretical arguments, metric extensions of GR also provide an appropriate tool for analysing gravity tests performed in the solar system. Most precise tests realized at or beyond the AU scale strongly rely on Doppler ranging, hence on an appropriate modeling of electromagnetic links and the trajectories of massive bodies. Metric extensions of GR provide a general and simple expression for the time delay function $\mathcal{T}(\mathbf{x}_1, \mathbf{x}_2)$ which describes the links used to follow a massive probe ($\mathbf{x}_a \equiv r_a(\sin \theta_a \cos \varphi_a, \sin \theta_a \sin \varphi_a, \cos \theta_a)$ with $a = 2, 1$ respectively denoting the coordinates of the probe and a station on Earth, $\mathcal{T}(\mathbf{x}_1, \mathbf{x}_2)$ is written here for a static isotropic metric (6))³⁸

$$c\mathcal{T}(r_1, r_2, \phi) \equiv \int_{r_1}^{r_2} \frac{-\frac{g_{rr}}{g_{00}}(r)dr}{\sqrt{-\frac{g_{rr}}{g_{00}}(r) - \frac{\rho^2}{r^2}}} \quad , \quad \phi = \int_{r_1}^{r_2} \frac{\rho dr/r^2}{\sqrt{-\frac{g_{rr}}{g_{00}}(r) - \frac{\rho^2}{r^2}}} \\ \cos \phi \equiv \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 \cos(\varphi_2 - \varphi_1) \quad (9)$$

ϕ is the relative angle, when seen from the gravitational source, of the two points \mathbf{x}_1 and \mathbf{x}_2 and ρ the impact parameter of the light ray joining these points. The two-point function $\mathcal{T}(\mathbf{x}_1, \mathbf{x}_2)$ describes the time taken by a light-like signal to propagate from position \mathbf{x}_1 to position \mathbf{x}_2 (thus giving a parametrization of lightcones). The time delay function can be seen to be parametrized by metric components (9), hence by the two gravitation potentials (Φ_N, Φ_P) . Doppler signals are obtained by taking the time derivative of $\mathcal{T}(\mathbf{x}_1, \mathbf{x}_2)$, and evaluating the latter on the trajectories of the probe and the Earth station. As geodesics must be determined according to the same metric extension of GR, the two potentials also enter the expressions of the trajectories²⁵.

Comparison between metric extensions and GR predictions can be performed explicitly and analysing the former within the framework of GR leads to deviations which take the form of Pioneer-like anomalies ($\delta a = \delta a_{\text{sec}} + \delta a_{\text{ann}}$ denotes the time derivative of Doppler signals⁹)

$$\delta a_{\text{sec}} \simeq -\frac{c^2}{2} \partial_r (\delta g_{00}) + [\ddot{r}_2]_{\text{GR}} \left\{ \frac{\delta(g_{00}g_{rr})}{2} - \delta g_{00} \right\} - \frac{c^2}{2} \partial_r^2 [g_{00}]_{\text{GR}} \delta r_2 \\ \delta a_{\text{ann}} \simeq \frac{d}{dt} \left\{ \left[\frac{d\phi}{dt} \right]_{\text{GR}} \delta \rho \right\} \quad (10)$$

The gravitational potentials in the two sectors contribute to both the secular part δa_{sec} and the modulated part δa_{ann} of the anomaly. These furthermore depend on the probe and Earth motions, which are obtained from the equations for geodesics and initial conditions. Hence, Pioneer-like anomalies appear as a prediction of metric extensions of GR. These moreover predict strong correlations between secular and modulated anomalies, which can be considered as signatures to be looked for in observations^{23,38}.

Besides directly, through a precise analysis of probe navigation data, the two gravitational potentials may also be expected to be determined as part of a big fit of all navigation and astrometric data, such as those used to obtain the ephemerides of planets and some of their characteristic constants. In such an approach, the two potentials play the same role as Eddington parameters β and γ ^{32,33,34}, with the additional feature of allowing significant dependences on length scales of the order of the solar system size²⁵. The results of Doppler and ranging observations should then be taken into account by using the time delay function (9) and the geodesics, depending on the two gravitational potentials (Φ_N, Φ_P) which define a general metric extension of GR. Clearly, the need to recall to numerical methods entails that the neighborhood defined by the two potentials, in their general form, is too large to be totally scanned by a fit. Hence, it appears crucial to design simplified models which depend on a small number of real parameters but still preserve the scale dependences which are most likely to be observed³⁹.

Metric extensions of GR also predict effects which can be expected to be exhibited by future experiments benefiting from a high increase in precision measurement. The time delay function (9) results in a particular scale dependence of the gravitational deflection of light which can be equivalently represented as an additional dependence of the deflection angle ω , or else of Eddington parameter γ , on the impact parameter of the light ray (M denotes the mass of the gravitational source, r its distance to the observer, χ the apparent relative angle between the light source and the gravitational source)²³

$$\omega(\chi) \simeq \frac{G_N M}{c^2 r} \frac{1 + \gamma(\chi)}{\tan \frac{\chi}{2}}$$

The two gravitational potentials characterizing metric extensions combine to induce a modification of the deflection angle which, in contrast to GR, contains a part which increases with the impact parameter. Such deviations should then become more noticeable for measurements performed with a high precision and at small deflection angles. In a near future, GAIA will perform a survey of our neighborhood in our galaxy and will follow with a very good accuracy an extremely large number of astrometric objects^{40,6}. This will include in particular a very large number of light deflection observations performed at small deflection angles, or at large angular distances from the Sun

$$\delta\omega < 40\mu\text{as}, \quad \omega \sim 4\text{mas}, \quad \chi \in [45^\circ, 135^\circ]$$

As a consequence, GAIA data will improve the accuracy for the observed mean value of γ (better than 2×10^{-6}) and will make it possible to map the dependence of γ on χ over its whole range of variation. Such a mapping could put into evidence small deviations from GR and moreover allow to determine and fit their particular dependence.

A definite answer to the question of modifying the gravitation theory at the solar system scale would be provided by missions embarking dedicated means for directly measuring the effects of gravity. A first example is OSS mission⁴¹ which, beside ranging facilities, will also possess a high precision accelerometer, thus allowing to distinguish the effects of gravitation from other forces affecting the probe and hence to determine unambiguously whether the probe follows a geodesic, and whether the latter corresponds to GR. Another mission, SAGAS⁴², aims at reaching the outer part of the solar system with, beside an accelerometer, an atomic clock on board. Using the combined information obtained, with a very high precision, from the optical links and the clock on board, one would be able to reconstruct the gravitational potentials in the two sectors, and thus to exactly determine the gravitation theory prevailing at the largest scales which can be reached by artificial probes.

5 Conclusion

When generalized under the form of a metric extension, GR remains a successful theory of gravitation within the whole solar system. Minimal modifications allow one to account for all gravity tests performed up to the solar system scale and to confirm the position of GR as the basis of gravitation theory. They moreover correspond to scale dependences of the gravitational coupling, thus bringing gravitation closer to the other fundamental interactions.

From a phenomenological point of view, metric extensions of GR appear as a convenient tool for testing gravity within the solar system. They may also provide a natural answer to the presence of anomalies when observations are analysed by confrontation with GR. The actual theory of gravitation can be approached by looking for such anomalies occurring in residuals of direct ranging data or big fits. It may also be determined by future high precision observations (GAIA) or dedicated missions in the solar system (OSS, SAGAS).

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